CRACK OPENING STRETCH

IN A PLATE OF FINITE WIDTH

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Abstract

The problem of a uniaxially stressed plate of finite width containing a centrally located "damage zone" is considered. It is assumed that the flaw may be represented by a part-through crack perpendicular to the plate surface, the net ligaments in the plane of the crack and through-the-thickness narrow strips ahead of the crack ends are fully yielded, and in the yielded sections the material may carry only a constant normal traction with magnitude equal to the yield strength. The problem is solved by neglecting the bending effects and the crack opening stretches at the center and the ends of the crack are obtained. Some applications of the results are indicated by using the concepts of critical crack opening stretch and constant slope plastic instability.

1. INTRODUCTION

In this paper we reconsider the problem of an infinite strip or a long plate of finite width with a symmetrically located crack perpendicular to the boundaries (Figure 1). The elasticity problem for this geometry has been considered before in various publications (e.g.,[1-4]. In a configuration such as the cracked strip, unless the material is extremely "brittle" (in the sense that it may rupture without undergoing appreciable inelastic deformations around the crack tips), for crack lengths of the order of the width of net ligaments it is clear that the elasticity solution may not be adequate either to describe the stress and deformation states in the strip or as a prediction tool in the related fracture study. On the other hand under "plane stress" conditions if the material undergoes plastic deformations with the plastic zone size around the crack

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tips of the order of crack length, there is at least a certain school of thought adhering to the notion that the concepts of the crack opening stretch or the plastic instability may be used for fracture characterization of the material.

Aside from the verification of the validity of these concepts and the experimental determination of the related strength parameter for a given material, in applications one of the important problems is, of course, analytically finding a reasonable estimate of the crack opening stretch or the value of the external load at plastic instability. Even though there is no widely accepted standardization, the strength parameter of the material, the so-called critical crack opening stretch, is defined as the relative displacement measured at the crack tip at the onset of rupture (through, for example, photographically recorded diamond indentation marks). As for the quantity representing the severity of the external loads, ever since the publication of Dugdale's work on the subject [5], the crack opening displacement calculated at the crack tip by using the conventional plastic strip model has been considered to be quite adequate.

In this paper we will mainly be interested in studying the effect of crack length-to-plate width ratio, a/h on the plastic zone size and the crack opening displacement, δ in plates of finite width. It will be assumed that the plate has either a symetrically located through crack or a "damage zone" which may be represented by a part-through crack (Figure 1). In considering the part-through crack problem the bending effects will be neglected and it will further be assumed that the net ligaments connecting the part-through crack to the plate surfaces as well as a narrow through the thickness strip in the plane and ahead of the crack is fully yielded (shaded area in Figure 1 b). After calculating δ as a function of a/h and the applied load, one may estimate the load carrying capacity of the plate based on a criterion of critical crack opening stretch or plastic instability. Such estimates are also given in the paper.

2. FORMULATION OF THE PROBLEM

The problem is solved by using the standard superposition technique and the method described in [3]. In [3] it was shown that the plane elastostatics problem of an infinite strip $-h<x<h$, $-\infty<y<\infty$, containing a crack along $-a<x<a$, $y=0$ may be formulated in terms of the following integral equation:

$$\int_{-a}^{a}\left[\frac{1}{t-x}+k(x,t)\right]G(t)dt= -\frac{1+\kappa}{4\mu}\pi\ p(x),\quad (-a<x<a) \tag{1}$$

where

$$G(x)= \frac{\partial}{\partial x}v(x,+0),\quad (-a<x<a) \tag{2}$$

is the crack surface displacement (in y-direction),

$$p(x)= -\sigma_{yy}(x,0),\quad 0=\sigma_{xy}(x,0),\quad (-a\ x\ a) \tag{3}$$

are the crack surface tractions which are assumed to be the only external loads acting on the strip, μ and κ ($\kappa= 3-4\nu$ for plane strain, $\kappa= (3-\nu)/(1+\nu)$ for the generalized plane stress, ν being the Poisson's ratio) are the elastic constants, and the kernel $k(x,t)$ is given by

$$k(x,t)= \int_0^\infty K(x,t,s)e^{-(h-t)}ds, \tag{4}$$

$$\begin{aligned}K(x,t,s)= e^{-hs}\{&-[1+(3+2hs)e^{-2hs}]\cosh(xs)\\ &-2xse^{-2hs}\sinh(xs) - [2xs\sinh(xs)\\ &+(3-2hs+e^{-2hs})\cosh(xs)]\,[1-2s(h-t)]\}/(1\\ &+4hse^{-2hs}-e^{-4hs}).\end{aligned} \tag{5}$$

The index of the singular integral equation (1) is +1, hence its solution is determinate within one arbitrary constant which is determined from the following single-valuedness condition:

$$\int_{-a}^{a} G(x)\,dx=0 \tag{6}$$

Once the density function G(x) is determined all the desired field quantities in the problem may be expressed in terms of definite integrals with G as the density and the related Green's functions as the kernels.

Referring to Figure 1 let 2h be the width of the plate, 2a and b the "equivalent" dimensions of the part-through crack representing the initial damage zone, b_o the plate thickness, a_p - a the size of the plastic zone in the plane of the crack, and σ_o the uniform tensile stress acting on the plate away from the crack region. It will be assumed that in addition to the "narrow strips" of length a_p- a in the plane and ahead of the internal crack, the net ligaments connecting the crack to the plate surfaces are also fully yielded. Thus, the yielded zones are shown in Figure 1 as the shaded regions. In formulating the problem it will be assumed that the damage zone can be approximated by a rectangular part-through crack of dimensions 2a and b with sides parallel and perpendicular to the plate surfaces, and the bending effects arising from the nonsymmetric orientation and shape of the part-through crack is negligible. The solution of the problem may then be obtained by the superposition of solutions of the following three problems:

Problem A: No crack; external load:

$$\sigma_{yy}(x,\mp\infty) = \sigma_o;$$

Problem B : Crack : $-a_p<x<a_p$, $y=0$;

external load : $\sigma_{yy}(x,0)=-p(x)=-\sigma_o$, $(-a_p<x<a_p)$;

Problem C : Crack : $-a_p<x<a_p$, $y=0$;

external load : $\sigma_{yy}(x,0)=-p(x)=\sigma_Y$

for $a<|x|<a_p$, $\sigma_{yy}(x,0)=-p(x)=\sigma_Y\frac{b_o-b}{b_o}$

for $-a<x<a$.

Here the dimensions b_o, h, b, a (Figure 1), the external load σ_o, and the yield strength σ_Y are known. The fictitious crack length $2a_p$ is unknown and is determined from the condition of finiteness of stress state at the crack tips $\mp a_p$.

Note that the problem has a symmetry in loading and geometry with respect to y axis and the Problem A has no contribution to the stress singularities. Therefore, the condition giving a_p may be expressed as

$$k_B+k_C=0 \tag{7}$$

where k_B and k_C are the stress intensity factors at a_p obtained from the solutions of Problems B and C, respectively and may be expressed in terms of the unknown density function G(x) as follows:

$$k_j=-\frac{4\mu}{1+\kappa}\lim_{x\to a_p}\sqrt{2(a_p-x)}\,G_j(x),\quad (j=B,C). \tag{8}$$

If we define the following dimensionless parameters

$$\lambda=a/h,\ \lambda_p=a_p/h, \tag{9}$$

equasion (7) may be written as

$$k_B+k_C=\sigma_o K_B(\lambda_p)-\sigma_Y K_C(\lambda_p,\lambda)=0. \tag{10}$$

The problem is solved simply in an inverse manner, i.e., for a fixed λ_p, λ is varied, K_B and K_C are found (for unit loads) from (1) and (8), the corresponding load ratio σ_o/σ_Y is found from (10), and then the curves giving $(a/a_p)=(\lambda/\lambda_p)$ vs σ_o/σ_Y are prepared with λ_p as a parameter.

After determining the density function

$$G(x)=G_B(X)+G_C(x) \tag{11}$$

The crack surface displacement may be evaluated from

$$v(x,+0)= -\int_x^{a_p} G(x)\,dx. \tag{12}$$

From (12) the crack opening stretches of physical interest, i.e., that at the actual crack tip x=a, and at the center x=0 may be obtained as

$$\delta=v(a,+0)-v(a,-0)= -2\int_a^{a_p} G(x)\,dx, \tag{13}$$

$$\delta_o=v(0,+0)-v(0,-0)= -2\int_0^{a_p} G(x)\,dx. \tag{14}$$

3. THE RESULTS

The calculated results which are shown in Figures 2-11 are obtained for three values of relative area of the part-through crack, namely $(b/b_o)=0.5$, 0.75, and 1 (i.e., through crack). Figures 2-4 give the information to obtain the plastic zone size or a_p for a given load ratio σ_o/σ_Y and crack length, a. For $\lambda_p=1$ the crack plane is fully yielded and we have

$$\frac{\sigma_o}{\sigma_Y} =1- \frac{b}{b_o}\,\frac{a}{h} =1- \frac{b}{b_o}\,\frac{a}{a_p}, \tag{15}$$

giving the straight lines shown in the figures. The value of $\lambda_p=(a_p/h)=0$ corresponds to the infinite plane for which in the case of through crack $(b=b_o)$ we have (Figure 4)

$$\frac{a}{a_p}=\cos\left(\frac{\pi\sigma_o}{2\sigma_Y}\right). \tag{16}$$

The crack opening stretch δ calculated at the crack tip x=a (see equation 13) is shown in Figures 5-7. The normalization factor d used in these figures is given by

$$d=\frac{1+\kappa}{2\mu}\sigma_Y a=\frac{4\sigma_Y a}{E} \text{ (plane stress)}. \tag{17}$$

In the figures $\lambda=a/h$ is used as the parameter. The value $\lambda=0$ again corresponds to the infinite plane for which in the case of through crack we have (Figure 7)

$$\frac{\delta}{d}= - \frac{2}{\pi}\log\left(\cos\frac{\pi\sigma_0}{2\sigma_Y}\right). \tag{18}$$

The asymptotic values of the δ curves indicated in the figures are the load ratios corresponding to the fully-yielded net section and are given by

$$\frac{\sigma}{\sigma_Y}= 1- \frac{b}{b_0}\lambda. \tag{19}$$

Note that in limit when $\lambda=1$ the (δ/d) vs (σ_0/σ_Y) curve reduce to the straight line $(\sigma_0/\sigma_Y)= 1-b/b_0$ shown in the figures.

In the case of a part-through crack, from the view point of applications a more important quantity is the crack opening stretch δ_0 calculated at $x=0$, i.e., the maximum stretch. Figures 8 and 9 show this quantity for $(b/b_0)= 0.5$ and 0.75, respectively. The asymptotic values of these δ_0 curves too are given by (19).

If one adopts a critical crack opening stretch criterion for rupture then the load-carrying capacity of the plate may be obtained from

$$\delta_{max}= \delta_{cr}. \tag{20}$$

where δ_{max} is the measure of the intensity of the applied load with $\delta_{max}=\delta_0$ for the part-through crack and $\delta_{max}=\delta$ for the through crack. Figure 10 shows the load-carrying capacity of the plate with a through crack for values of critical crack opening stretch $0.1 \leq (\delta_{cr}/d) \leq \infty$. Note that for constant δ the derivative of

σ_o vs λ=a/h curve is negative, meaning that, according to this criterion the rupture process is unstable. Figure 10 is obtained from the calculated results giving Figure 7. For a plate with a part-through crack similar curves giving the load-carrying capacity may be obtained from Figures 8 and 9. The straight line shown in Figure 10 corresponds to the fully-yielded net section. In this case, since the material is assumed to have no strain-hardening, the corresponding value of the crack opening stretch is very large (theoretically, infinite).

It should be pointed out that the results found in this paper for plates with finite width are similar to those found in shells, λ=a/h playing the role of the shell parameter $\lambda=[12(1-\nu^2)]^{1/4}\cdot(a/\sqrt{Rb_o})$ (with R the radius of curvature, b_o the thickness). (see for example [6,7]). In studying the burst phenomenon in cylindrical shells, it was observed that one may also use the value of δ corresponding to a standard slope in δ/d vs σ_o/σ_Y plot rather than a fixed value of δ itself as the representative of the intensity of the applied loads [8]. The rational here being that, as seen from Figures 5-9, after reaching a certain value any further increase in the applied load may cause very large increases in the crack opening stretch which may be interpreted as the onset of "necking" process, hence plastic instability. Figure 11 shows the load-carrying capacity of a plate with a through crack obtained by using this concept for various standard slopes.

Finally it should be emphasized that in the model used in this paper the effect of the strain hardening has not been taken into account. Hence, in applications σ_Y of this paper should be considered as a "flow stress" rather than the standard 0.2 percent offset yield strength of the material. The value of the flow stress $\bar{\sigma}$ may be selected as either $\bar{\sigma}=(\sigma_Y + \sigma_u)/2$ or $\bar{\sigma}= (1 + \alpha)\sigma_Y$ where σ_Y is the standard yield strength, σ_u the ultimate strength and α a fixed parameter, $0<\alpha<(\sigma_u - \sigma_Y)/\sigma_Y$.

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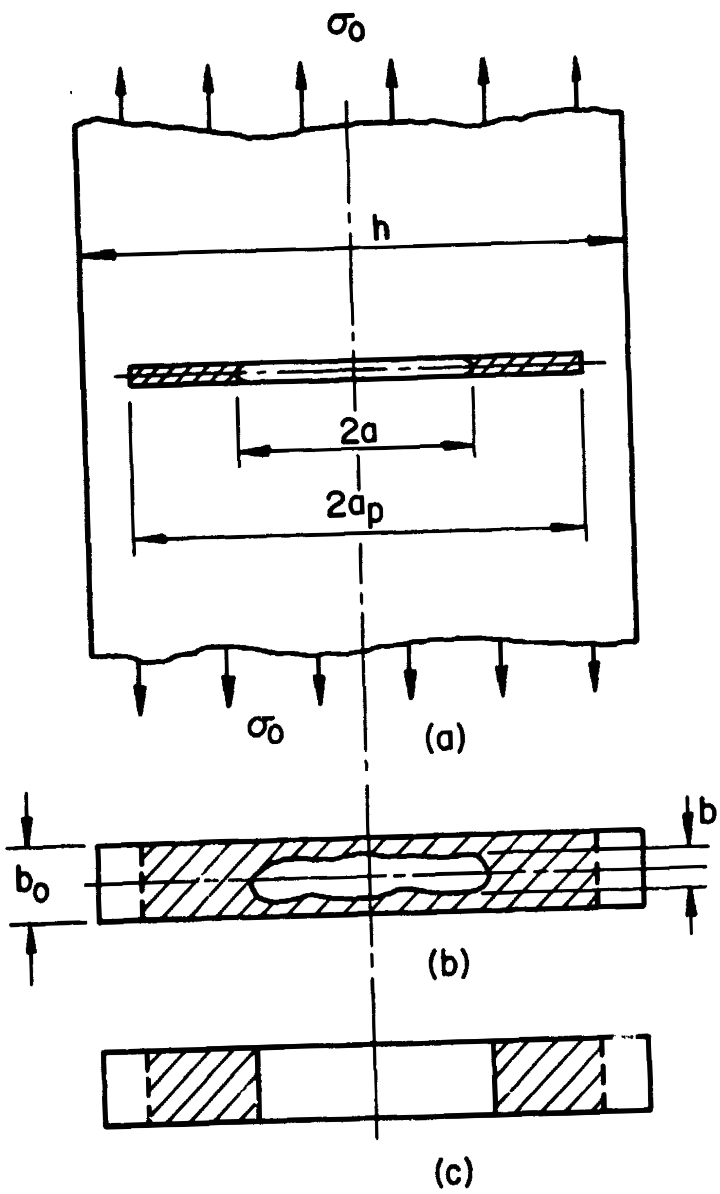

Fig. 1. Strip and crack geometry.

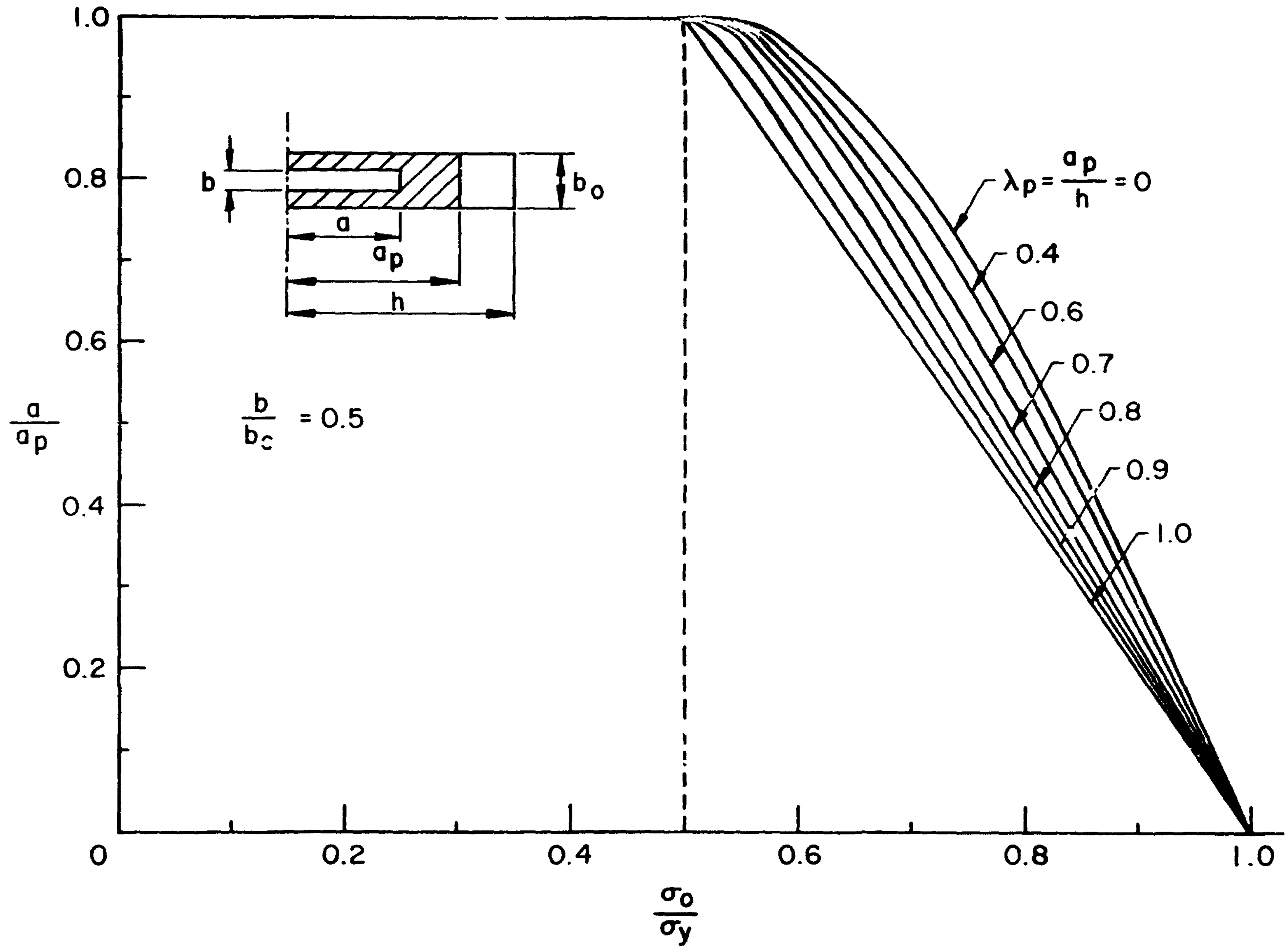

Fig. 2. Curves to be used to determine the plastic zone size a_p-a for crack depth ratio b/b_o=0.5.

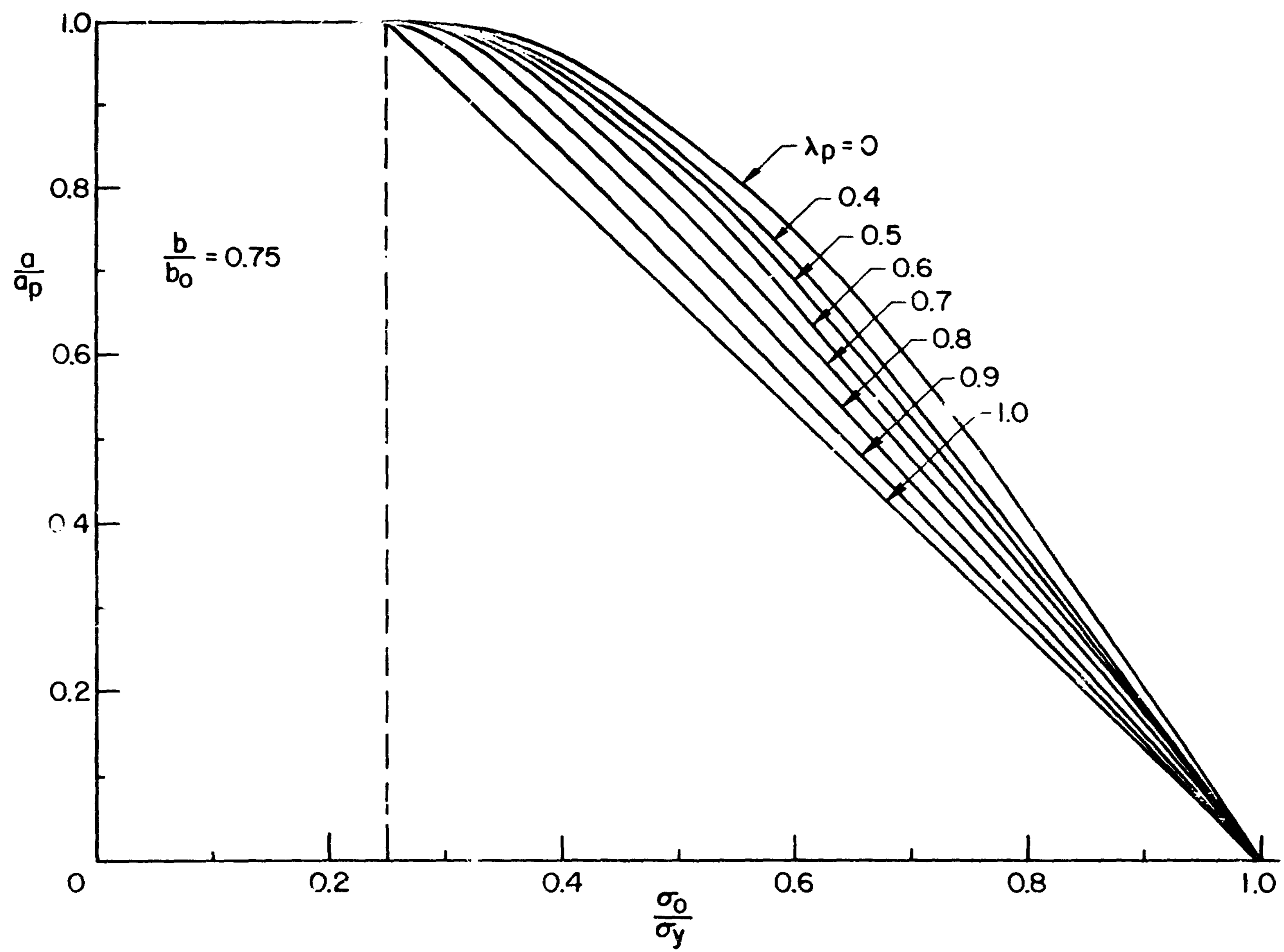

Fig. 3. Curves to be used to determine the plastic zone size $a_p - a$ for crack depth ratio $b/b_o = 0.75$.

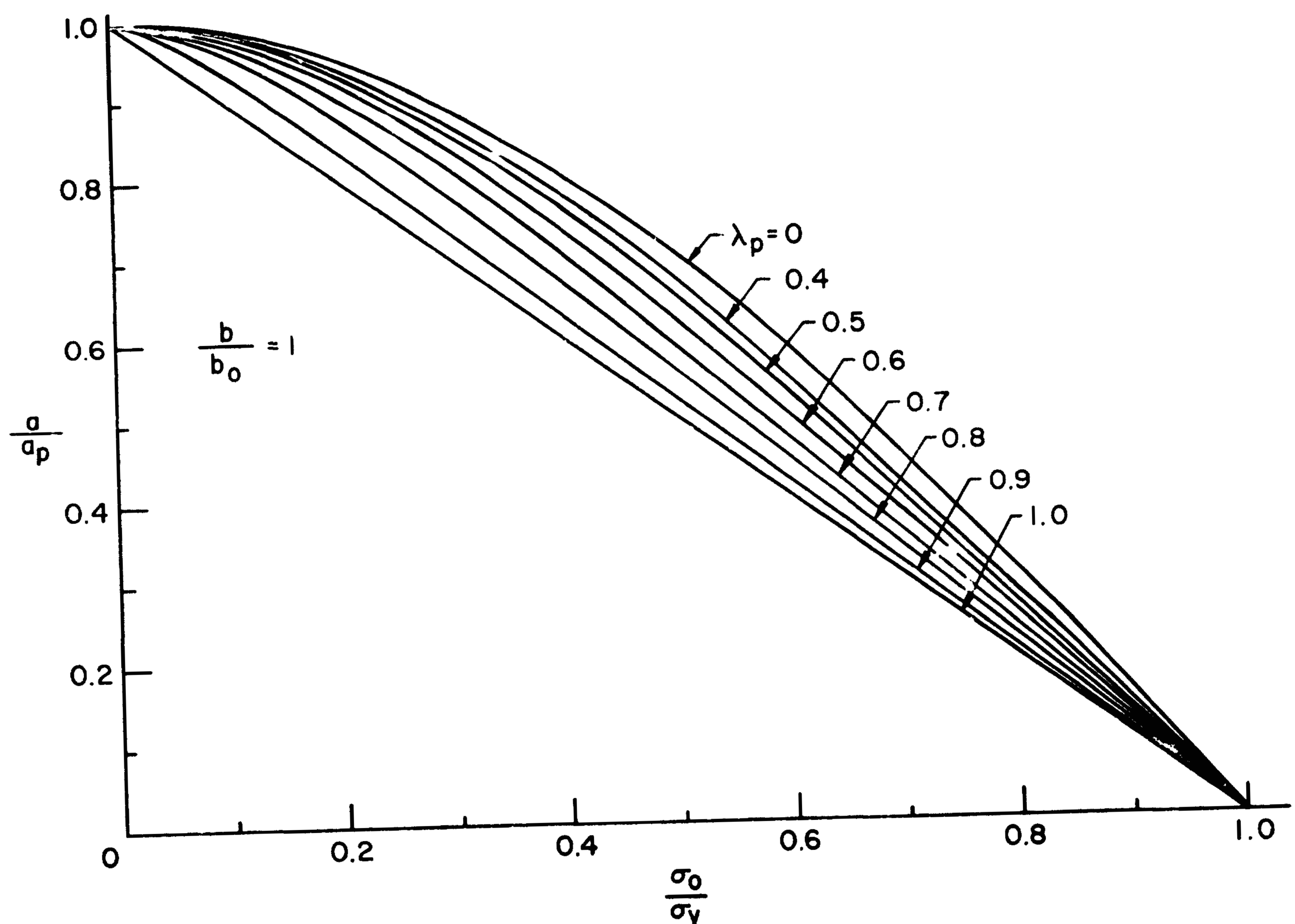

Fig. 4. Curves to be used to determine the plastic zone size $a_p - a$ for through crack.

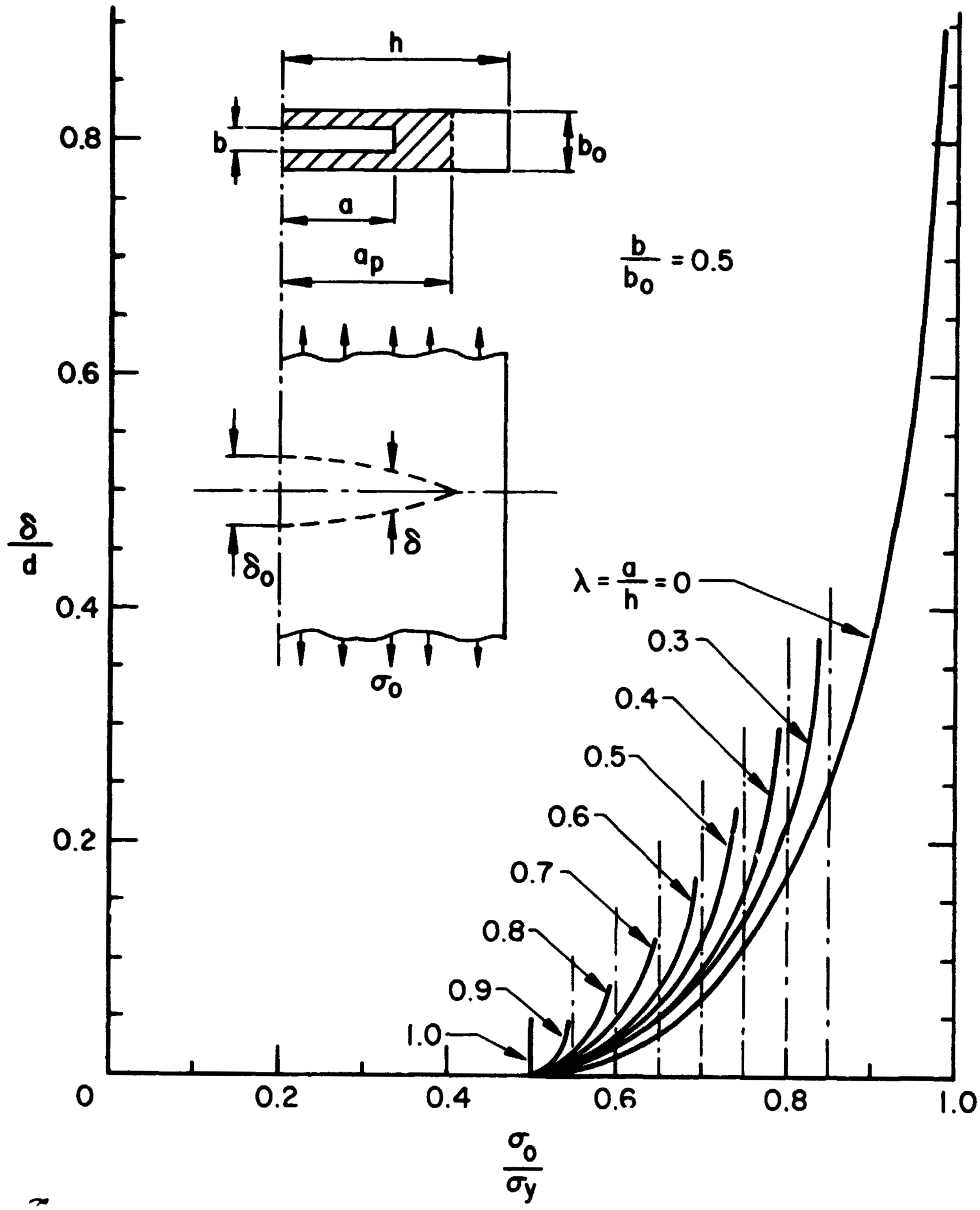

Fig. 5. The crack opening stretch δ at the crack tip for $b/b_o = 0.5$ ($d = 4\sigma_Y a/E$).

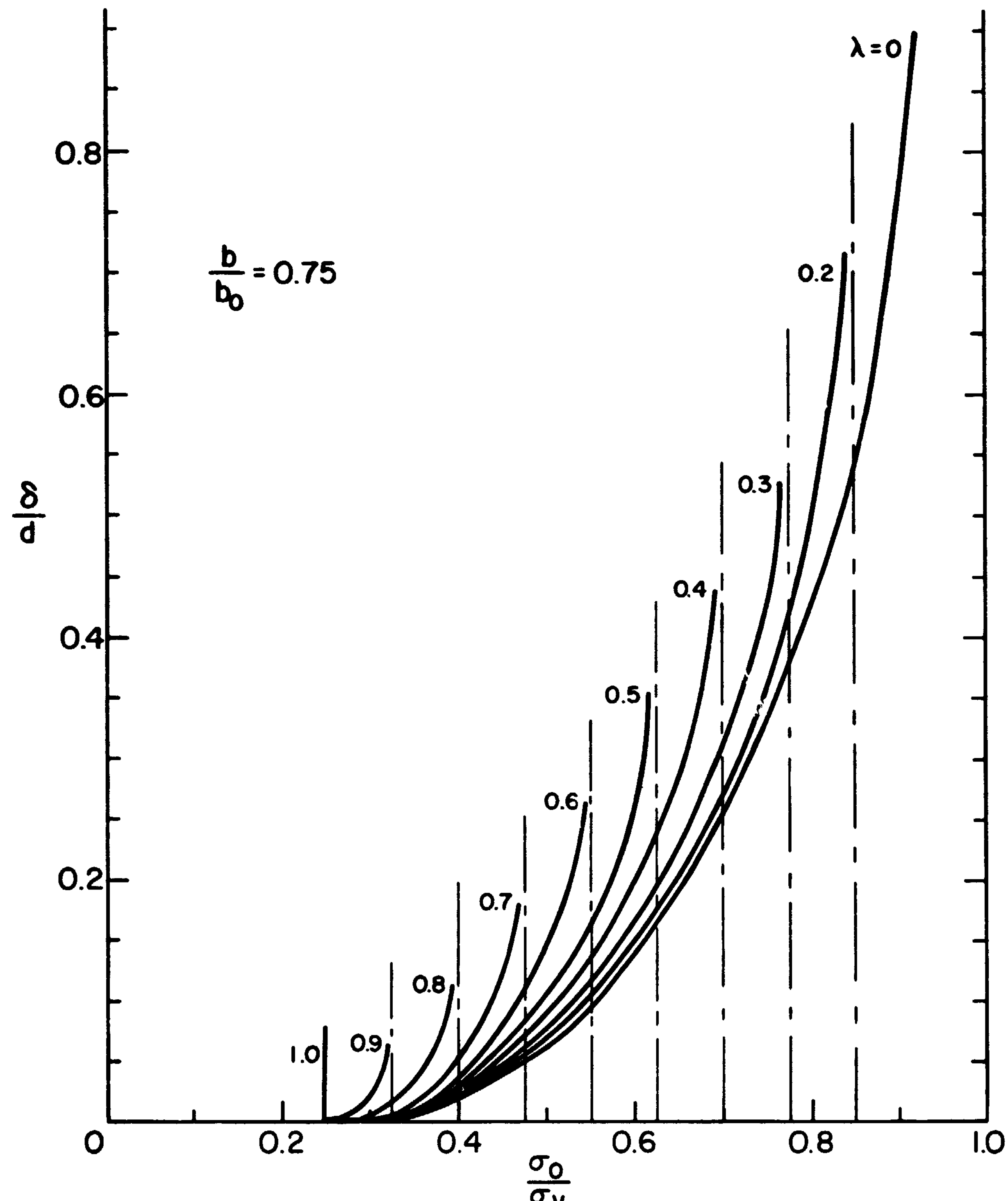

Fig. 6. The crack opening stretch δ at the crack tip for b/b_o=0.75 ($d=4\sigma_y\ a/E$).

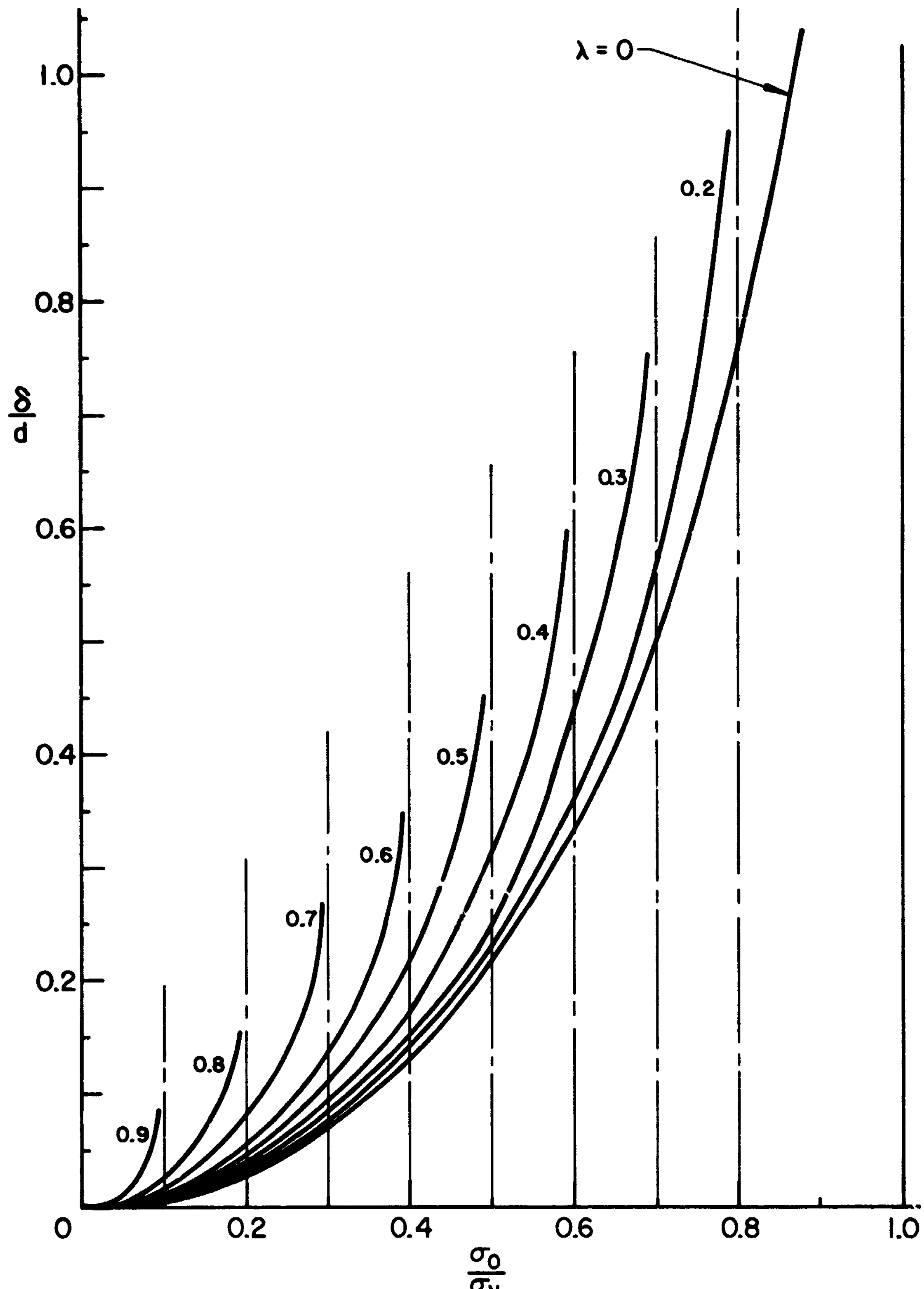

Fig. 7. The crack opening stretch δ at the crack tip for through crack ($d=4\sigma_Y a/E$).

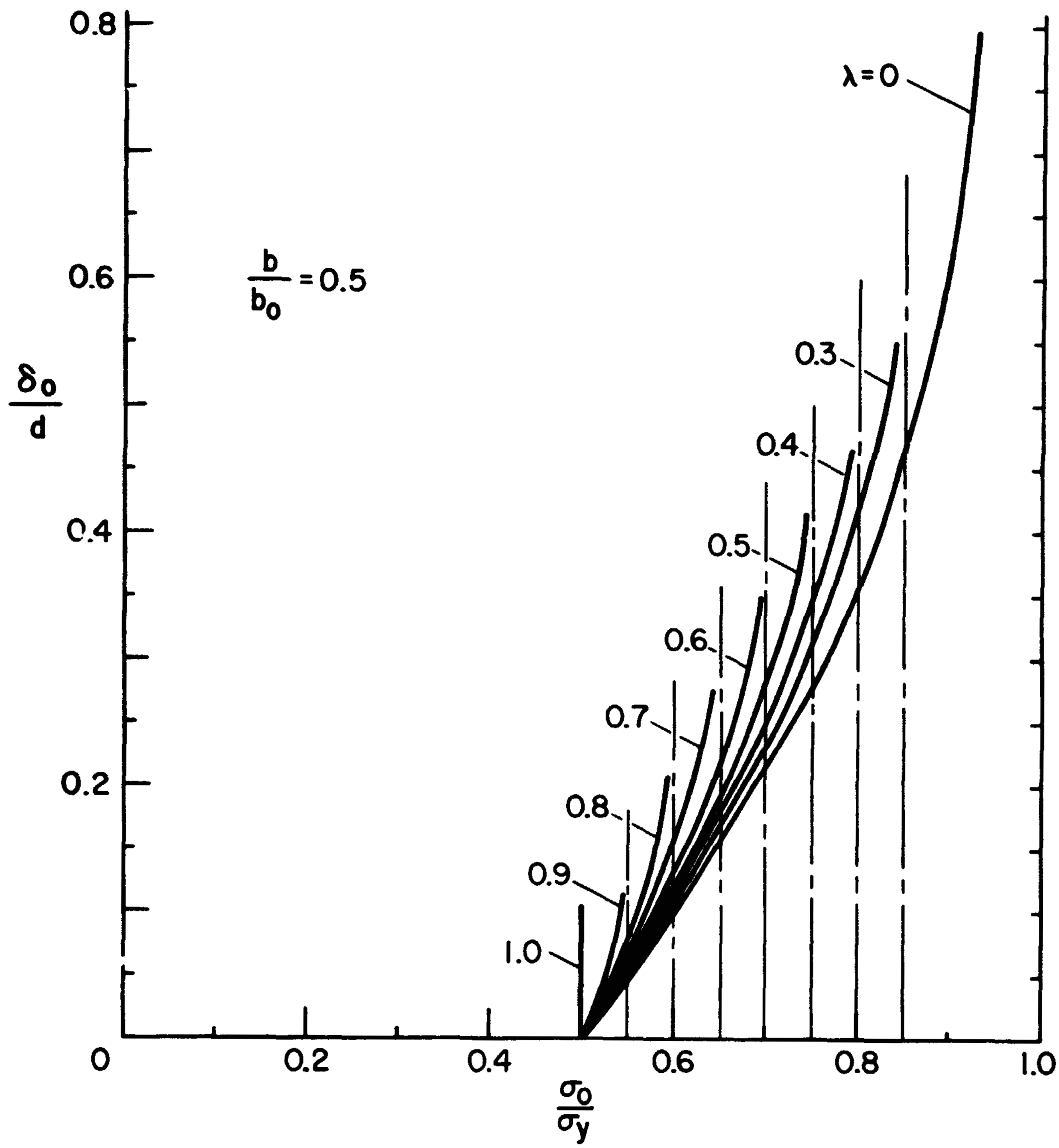

Fig. 8. The crack opening stretch δ_o at the center x=0 for $b/b_o=0.5$ ($d=4\sigma_Y\ a/E$).

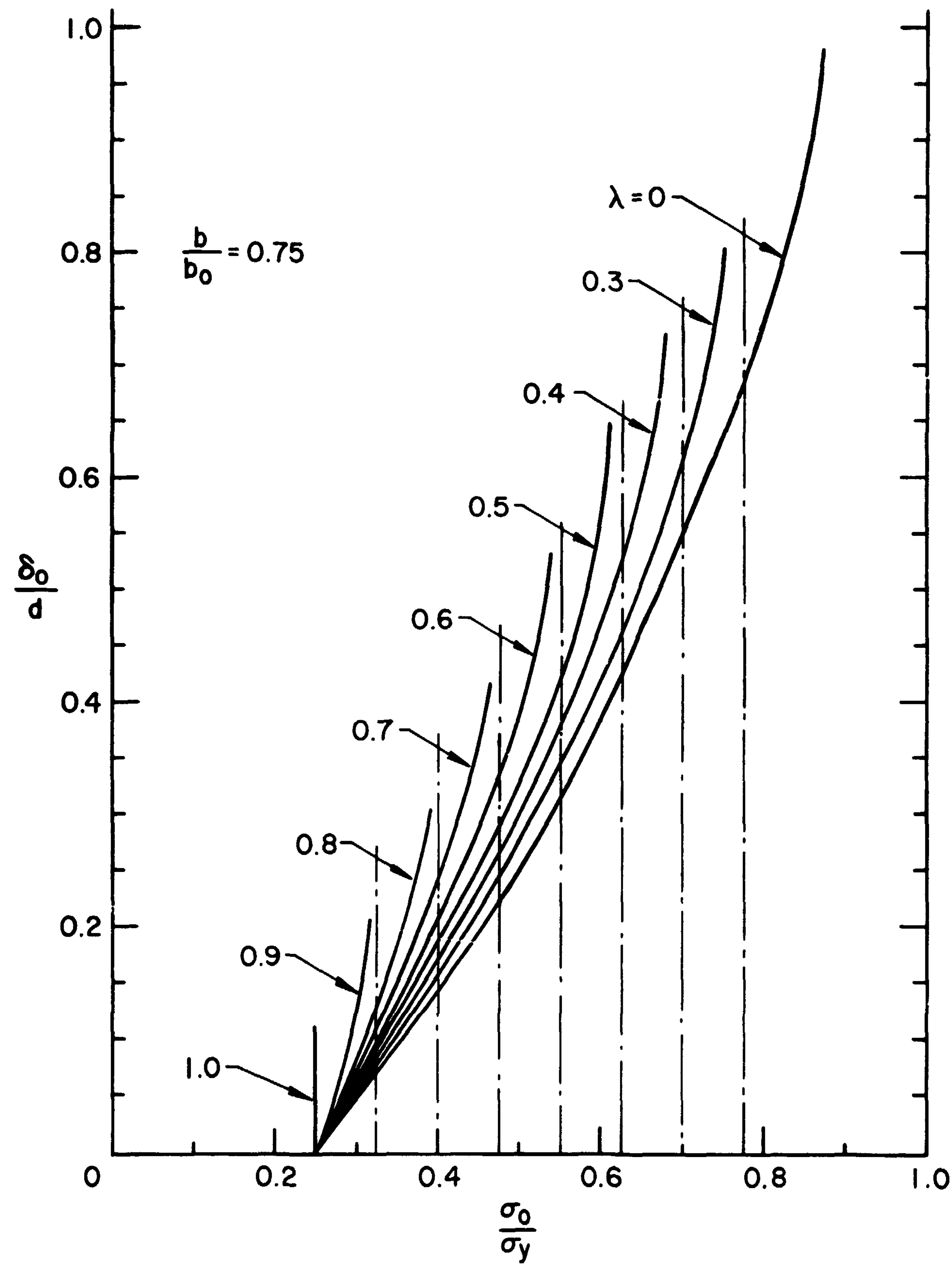

Fig. 9. The crack opening stretch δ_o at the center x=0 for b/b_o=0.75 ($d=4\sigma_Y a/E$).

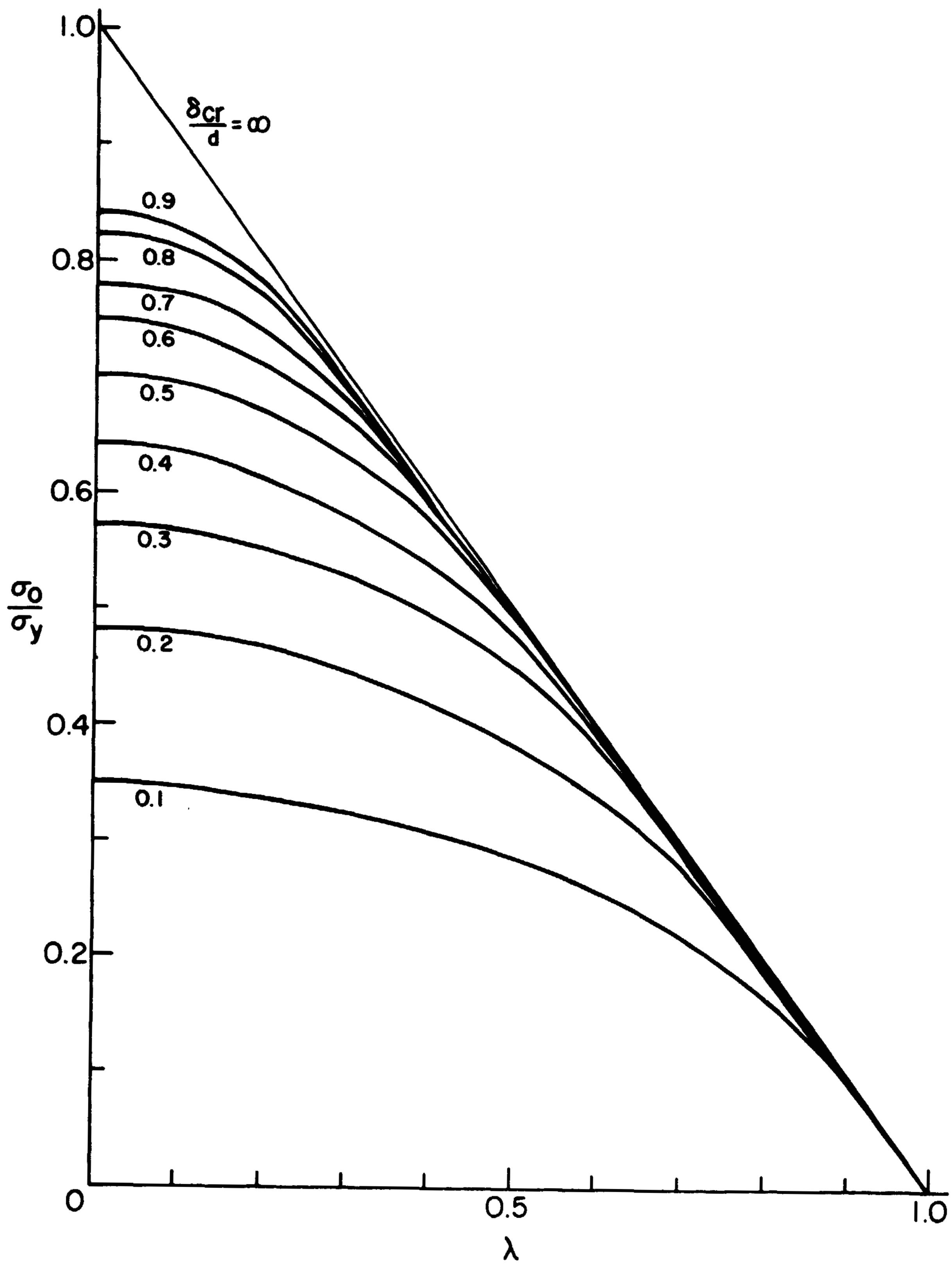

Fig.10. Load carrying capacity of a plate with finite width 2h and containing a through crack of length 2a based on critical crack opening stretch criterion ($\lambda = a/h$).

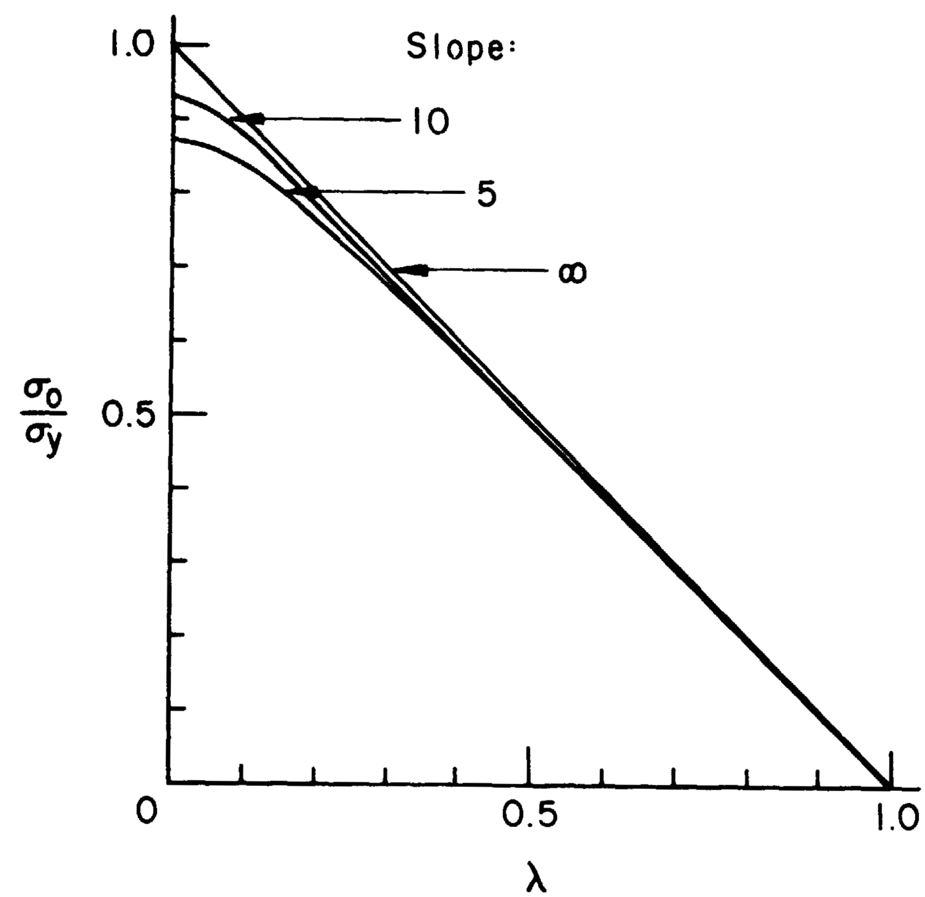

Fig.11. Load carrying capacity of a cracked plate based on constant slope - plastic instability ($\lambda = a/h$).